Digital PID Controller Design for DC-DC Buck Converter

Thesis submitted in partial fulfillment for the award degree of

Master of Technology

in

Electrical Engineering
(Specialization: Control & Automation)

by

Ashis Mondal

Department of Electrical Engineering

National Institute of Technology, Rourkela.
May- 2014

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Certificate

This is to certify that the report entitled, "**Digital PID controller Design for DC-DC Buck Converter"** submitted by **Ashis Mondal** to the Department of Electrical Engineering, National Institute Of Technology, Rourkela, India, during the academic session 2013-2014 for the award of the degree of **Master of Technology** in **"Control & Automation"** specialization, is a bona-fide record of work carried by him under my supervision and guidance. The thesis has fulfilled all the requirements as per the regulations of this institute and in my opinion reached the standard for submission.

……..……..……....
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Place : Rourkela, India
Date: May, 2014

Declaration

I, **Ashis Mondal**, declare that:

1. The work contained in this thesis is original and has been done by me under the guidance of my supervisor Dr. Susovon Samanta
2. The work has not been submitted to any other Institute for any degree or diploma.
3. I have followed the guidelines provided by the institute in preparing the report.
4. I have conformed to the norms and guidelines given in the Ethical Code of Conduct of the Institute.
5. Whenever I have used materials (data, theoretical analysis, figures and text) from other sources, I have given due credit to them by citing them in the text of the report and giving their details in the references.

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.....
(Ashis Mondal)

Acknowledgement

First of all I like to thanks from deep of my heart to my supervisor Dr. Susovon Samanta for the confidence that he accorded to me by accepting to supervise this thesis. I express my warm gratitude for his precious support, valuable guidance and consistent encouragement throughout the course of my M. Tech.

My heartiest thanks to Dr. Bidyadhar Subudhi, an inspiring faculty and Dr. A. K. Panda, Head of Department of Electrical Engineering, NIT, Rourkela.

Special thanks to Green, Gour, Gaurav for their support, care and love. I would like to extend special gratitude to my friends in NIT Rourkela, without whom, this journey would not have been this enjoyable.

Finally, I dedicate this thesis to my family: my dear father, my dearest mother who supported me morally despite the distance that separates us. I thank them from the bottom of my heart for their motivation, inspiration, love they always give me. Without their support nothing would have been possible. I am greatly indebted to them for everything that I am.

Abstract

DC-DC converters with computerized digital control methods picked up ubiquity because of their high productivity, low power utilization, higher resistance to natural changes, for example, temperature and maturing of parts, capacity to interface effortlessly, of programmability and to actualize advanced control plans. Their requisitions incorporate compact electronic gadgets, for example, computer and smart phones others.

Various techniques have been developed to meet the requirement of output voltage and at the same time it is also necessary to get more resolution to increase precision. The other is to develop new control methods that can utilize the advantages of the digital controller so as to improve the dynamic performance of the switching power converters.

The objective of this thesis is to study current techniques of DPWM generation and to develop new techniques using 8052 for low cost implementation and PID controller implementation. Usage of computerized PID controllers for DC-DC converters is talked about in this thesis. The essential preferences of digital control over analog control are higher invulnerability to ecological changes, for example, temperature and maturing of parts, expanded adaptability by changing the product, more praiseworthy control procedures and lessened number of segments. Simple PID controller was initially planned utilizing frequency response techniques, and then changed over into digital control.

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Chapter 1

Introduction

1.1 Introduction

The electronics industry, the fastest growing industry in this planet and would be remain so due to advancement and revolution in VLSI technology. This digital technology makes life of people very comfort and easy, and because of this Digital technology become most popular among all. Nowadays without electronics gadget like computer, cell phone, digital sound system, home appliances our life become like hell. That is why the electronics generating maximum revenue and growing faster with the new technological advancement. As the application of different devices is different, the power supply, current and voltage rating would be different as per as their specification. For that for that we need fixed regulated power supply. There is two type of voltage regulation method in general.

- Linear Regulator
- Switching Regulator

1.1.1. Linear Regulators

This regelation is done by using variable resistor. It is made by connecting a variable resistor series with the load resistor and the change in the value of resistance is done as per as the load. The advantage of this type of regulator is that it is simple and less costly but it is less efficient due power dissipation in the resistor has made unpopular. The regulation can also be done with ZENER diode but major problem is less efficient.

1.1.2 Switching Regulators

The switching regulator is become very popular nowadays because of its high power conversion efficiency. The efficiency can be achieved by the use of switching regulator up to 90%. This type of regulator also has great design flexibility. A switching regulator generally use a power switch by turning on & off it transfer energy partly from source to the load. This switch is controlled by controller and that is responsible for desire output. As it is using the controlled switch to regulate the power supply it is called switching regulator.

The loss of energy in this scheme is very less because of absence of resistor .this reason lead the switching regulator more efficient as compared to linear regulator. Also the output voltage of this regulator does not depend upon the load.

There is various type of switching regulator but our discussion is limited to Buck Regulator. The schematic diagram of buck converter is shown in Fig. 1.3.As this converter uses two power switch an operated synchronously i.e.one after another it is called synchronous Buck regulator. Buck converter, one type of step down chopper where output voltage is less than the input voltage. Here the regulation is done by using simple switches (with ideally no on resistance or very low on resistance). These switches goes ON and OFF at a fixed rate called as switching frequency, to keep the output at desired level.

The output of DC-DC buck converter is given as

$$V_o = D \times V_{in} \tag{1.1}$$

Where, D is duty ratio is defined by the ratio of on time of the switch (T_{ON}) to the operating time period (T)

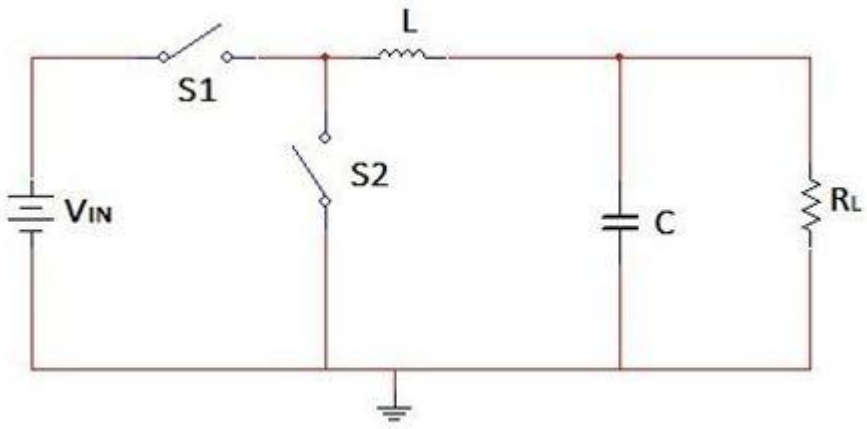


Figure 1 Synchronous buck converter

From the equation no. 1.1 we can say that the output voltage is the function of T_{ON} when the switching period T is fixed ,Thus by controlling the switch by controlled PWM signal we can control the output voltage.

1.2 Digital Controllers

Various type of controller is used to control the switch such as analog, digital, fuzzy etc. among these controllers Digital controller is very popular nowadays. There is plenty advantage digital control has over analog controller such as it is less sensitive to environmental changes, like change in temperature, humidity etc.it is more robust than analog controller as it is less sudden small short time change in the system. Another most important advantage is that if there is any change specification we don't need to change the component like analog controller, just

we need to change the program so, flexibility is greater than analog controller. It is also very compact and requires less area and mostly it is not very costly. But nothing is perfect in the world the digital controller has many advantage too. Such as there is always a delay incorporated with the system due to sampling process that makes the system nonlinear. Fig 1.4 shows the block diagram of digitally controlled synchronous buck converter operating in Voltage Mode Control.

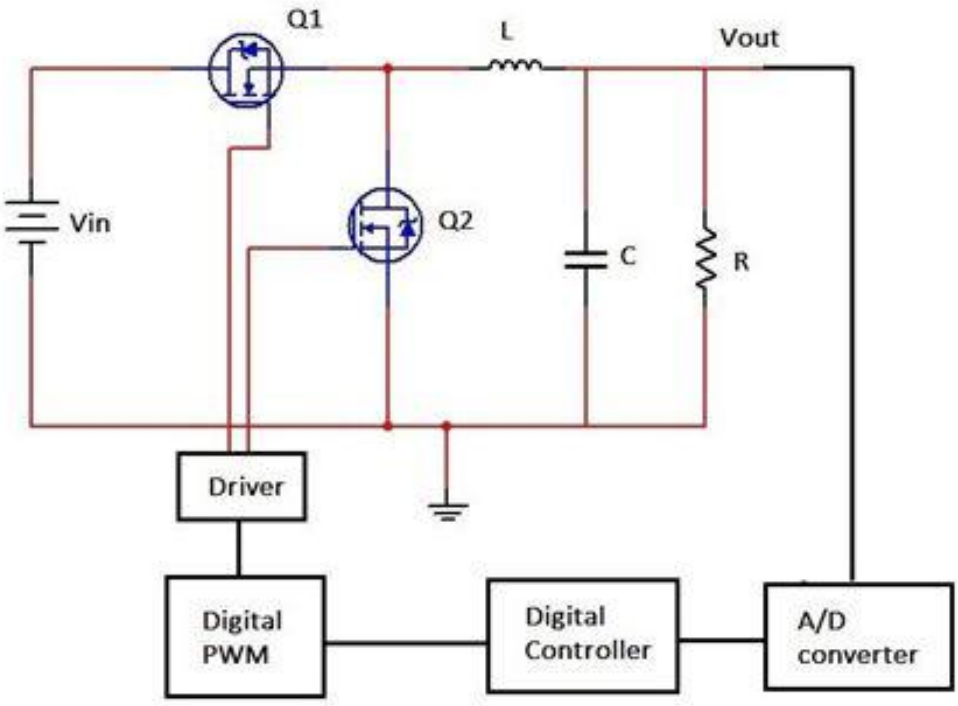


Figure 2 block diagram of closed loop synchronous buck converter

1.3 Objective

In this thesis various DPWM technique using different microcontroller are studied and various controlling method of DPWM signal is implemented experimentally. the main aim of this project is to design low cost ,efficient and convenient controlled DPWM technique using various type of microcontroller such as 8052, Audrino etc. This thesis is also meant to compare the designed method with the existed methods to checks its efficiency and performances.

1.7 Thesis Structure

- In order to analyze and check the performance of DPWM generation techniques, for a DC-DC converter the proper design of power stage parameters of DC-DC buck converter are necessary. Chapter 2 discusses designing of parameters viz. L, C, R of DC-DC buck converter.
- Chapter 3 presents the principle of Pulse Width Modulation (PWM). It also discusses present techniques of PWM generation in analog as well as digital domain. In this chapter the basics of microcontroller and its way of exploitation for PWM generation also included.
- Chapter 4 presents the PID controller design for controlling the voltage of the SBC and its hardware implementation.
- At last Chapter 5 contains conclusion to the thesis. Scope of future work extending the study further is suggested in this chapter.

Chapter 2

Synchronous Buck Converter design

This chapter deals with the design of synchronous buck converter. By this chapter we are able to describe how to select the inductor and capacitor, various problems for designing Buck Converter.

2.1 Synchronous Buck converter

Figure 2.1 shows the general configuration of synchronous buck converter. It uses two switches which is nothing but a MOSFET or we can use IGBT .In this thesis we use MOSFET to design the Buck converter. In some buck converter the low side MOSFET Q2 is replaced by schottkey diode or any general purpose diode such type of converter is called asynchronous Buck convereter

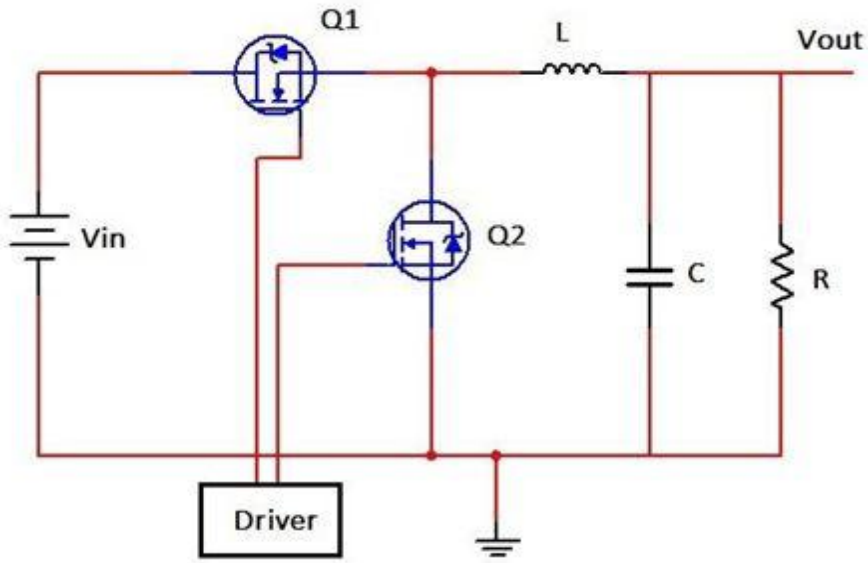


Figure 3 Synchronous buck converter with driver

To determine the proper value of inductor and capacitor following parameter information are nessecery.

1. Applied DC voltage: V_{IN}
2. Nominal output voltage: V_{OUT}

From fig 2.1 it can be conclude that it works like LC filter like that receives a voltage square wave at its input which is producing by controlled switching action of two MOSFET and after proper filtering produces a regulated output voltage V_{OUT} .

2.2 Inductor selection

The fundamental property of an inductor is to oppose the change in the magnitude of current passing through it. In buck converter the switching action of MOSFET is done at very high speed. Hence switch produces discontinuous output current, but it is the inductor, which overcome this problem. During the ON state of MOSFET an electric current flows in the circuit and energy is stored in the inductor (charging). As soon as the switch is turned off, there are no current flows to the circuit. At this time inductor released the entire energy which is stored in ON time. At steady state condition**,** the average inductor current I_L is equal to the output current $I_{avg.}$

Figure 2.2 shows the inductor current vs. time in CCM where the inductor current never reached to its zero value. It can be seen from the figure, the inductor current is not constant, but varies around I_{avg} between a maximum value I_{max} and a minimum value I_{min} whose difference ΔI_L is the peak-to-peak inductor current ripple.

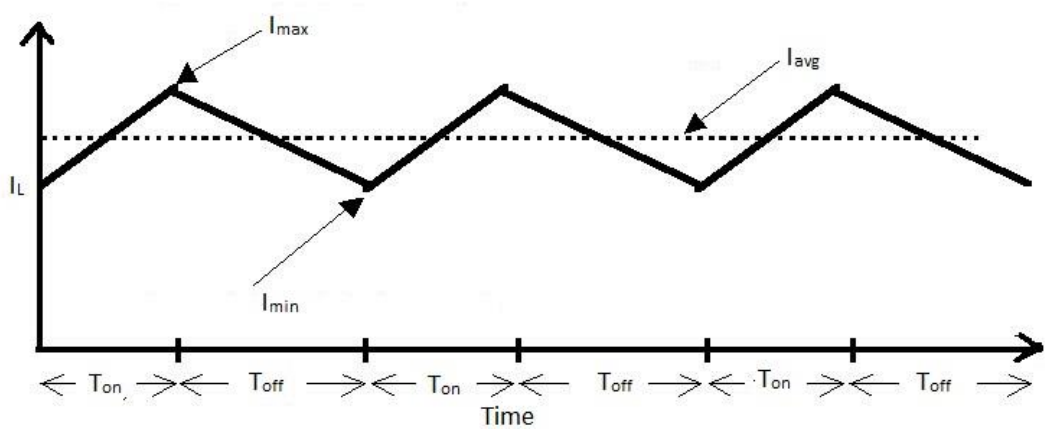


Figure 4 inductor current vs. time(t) in CCM

For choosing the value of inductor key factor need to be concern that the peak to peak ripple current ΔIL of inductor should not cross the permissible value there is a thumb rule that the peak to peak inductor ripple current ΔIL should be around 30% to 40% of average inductor current $Iavg$ or output current.

$$L = \frac{(V_{IN} - V_{out})V_{out}}{V_{IN} F_{SW} \Delta I_L} \qquad (2.1)$$

Where

- V_{IN} Input voltage (in V)
- V_{OUT} Output voltage (in V)
- F_{SW} Switching frequency (in Hz)
- L Inductance (in H)
- ΔI_L Peak-to-peak inductor current (in A)

When selecting a value of inductor three is another parameter need to be consider , that is the saturation current I_{SAT} of the inductor, which should never be exceeded in the operation. Operating the inductor above I_{SAT} would cause a significant inductance loss and a steep increase of the inductor current during the charging phase.

2.3 Selecting the Capacitor

The main function of capacitor is to maintain constant output voltage.it makes the output voltage ripple free. As ideal capacitor is almost practically impossible to construct there always an ESR incorporated with it and this ESR affects the output voltage. The best practice is to use low-ESR capacitors to minimize the ripple on the output voltage.

For a certain peak-to-peak output voltage ripple ($\Delta V_{OUT,RIPPLE}$), the required maximum ESR of the output capacitor can be calculated by using the following equation [5]

$$ESR = \frac{\Delta V_{OUT,RIPPLE}}{\Delta I_L} \tag{2.2}$$

Where, ΔI_L is the inductor peak to peak ripple current

The equation for calculating the value of capacitor is as follows [5],

$$C = \frac{L\, I_{L,max}^2}{(V_{OUT}+\Delta V_{OUT,OVERSHOOT})^2 - V_{OUT}^2} \tag{2.3}$$

Where

- $\Delta V_{OUT,OVERSHOOT}$: Peak overshoots voltage allowed on the output.
- $I_{L,max}^1$: Peak inductor current.

Because of this ESR in the capacitor the loss of power takes place and due to this the capacitor gets heated up. This temperature rise negatively affects the lifetime & reliability of the capacitor. Since excessive temperature negatively affects the reliability and the lifetime of a capacitor, an output capacitor with an adequate current rating should be selected.

In order to achieve better output voltage regulation, low-ESR capacitors are required. Ceramic capacitors generally have very low ESR.

2.4 Selecting the MOSFET

Figure 2.1 shows the power stage of DC- DC Converter. The MOSFET Q1 High Side MOSFET, Q2 is the Low Side MOSFET. Basically both MOSFETs have to withstand the input voltage. The MOSFETs also have to have a capability to handle additional voltage spikes caused by parasitic inductances. The maximum current seen by both MOSFETs is the output current plus 50 % of the ripple current. Since both MOSFETs are switched dynamically their power dissipation results partly out of the static losses contributed by the on resistance and the current and partly out of the switching losses [6]. The MOSFET should be chosen which satisfy above criteria the voltage and power rating of MOSFETs can be obtained from its datasheet.

Once the power and voltage rating criteria is satisfied and $Vout$, load current , switching frequency etc. determine the operating condition. The selection of the MOSFET is as follows:

- Lower value of the RDS (on) have less power dissipation better converter will work.

- The rise and fall time for MOSFET. By the thumb rule, the time required to on & off a MOSFET, should be 100 times less than the switching period of the converter.

2.5 Driving of the MOSFET

One of the most crucial parts of the designing buck converter is to driving the MOSFET. From Fig 2.1 it is easily seen that the source of low side MOSFET Q2 is the grounded but the source of high side MOSFET Q2 is connected with the inductor or it can be said that the source Q2 is floating in this case because of this the gate voltage of high side MOSFET is required 10V - 15V higher than supply DC voltage (V_{IN}).

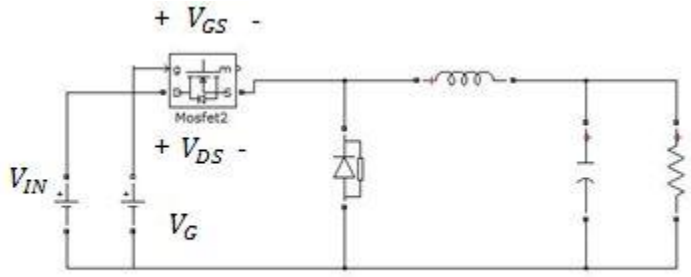


Figure 5 Buck converter

By using KVL ,

$$-V_{IN} - V_{GS} + V_{DS} + V_G = 0 \quad (2.4)$$

$$\text{Or,} \quad V_G = V_{IN} + V_{GS} - V_{DS} \quad (2.5)$$

Hence an extra driver circuitry is need to provide this V_G to high side MOSFET.one of the most popular driver IC is used as a driver in Buck Converter is IR2110 [6].

A bootstrap capacitor is connected between pin no. 7 & 6. During the off time of high side MOSFET Q2, the low side MOSFET is on, the bootstrap capacitor gets charged. During the on time of high side MOSFET Q2, the low side MOSFET is off. The bootstrap capacitor discharged through and desired MOSFET gate voltage is obtained [7]

For proper driving of the MOSFET the value and the type of the bootstrap capacitor must be chosen carefully [8].

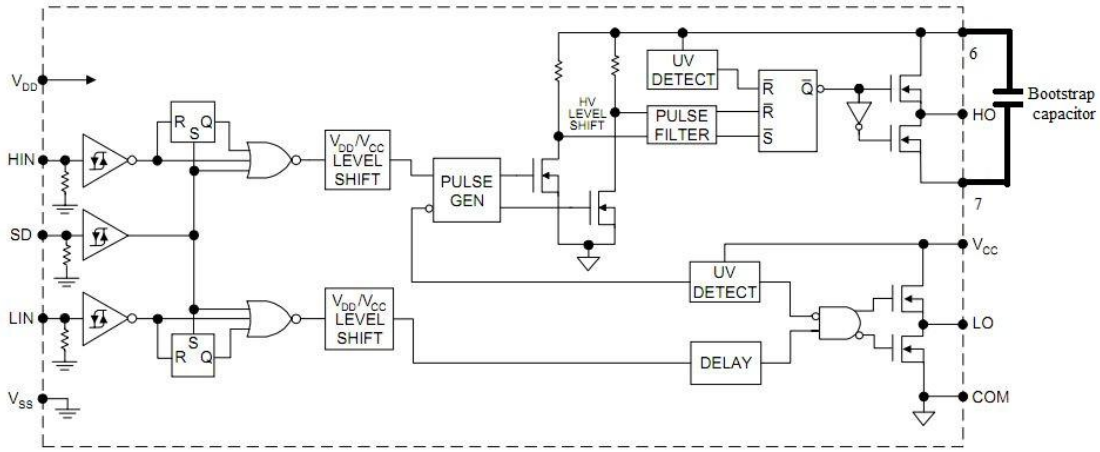


Figure 6 IR2110 IC with bootstrap capacitor

2.5 Simulation of the synchronous Buck Converter

The PSICE simulation model of synchronous buck converter and the corresponding result is shown in respective Fig.

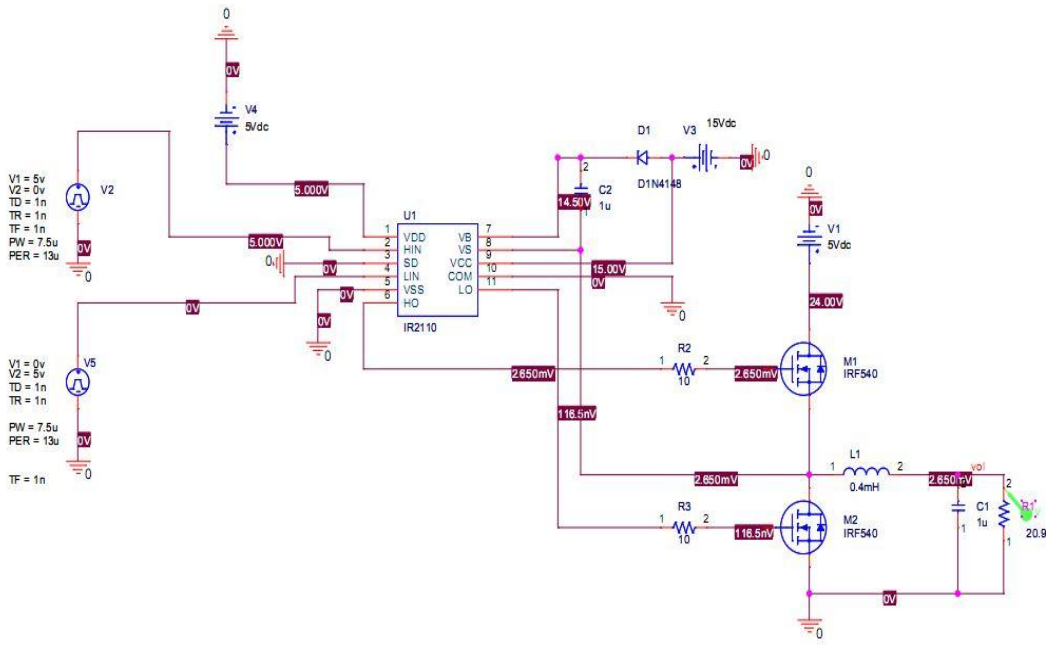


Figure 7 Simulation model of the synchronous Buck converter

2.6 Simulation Result

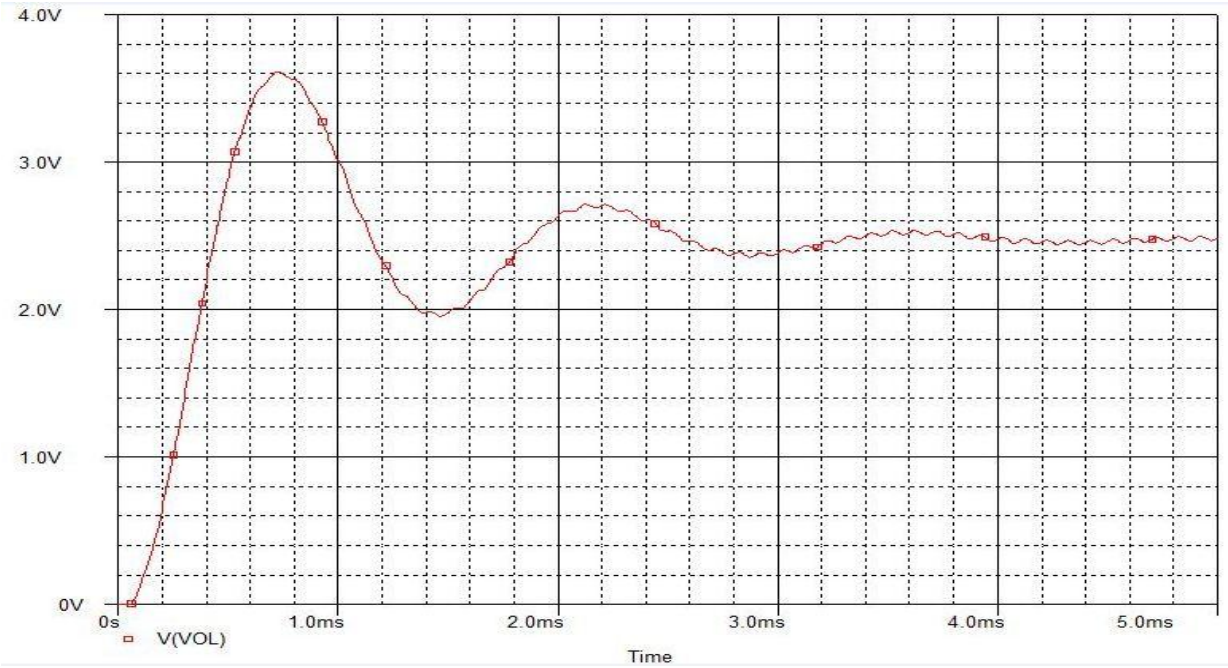


Figure 8 Simulation model of the synchronous Buck converter

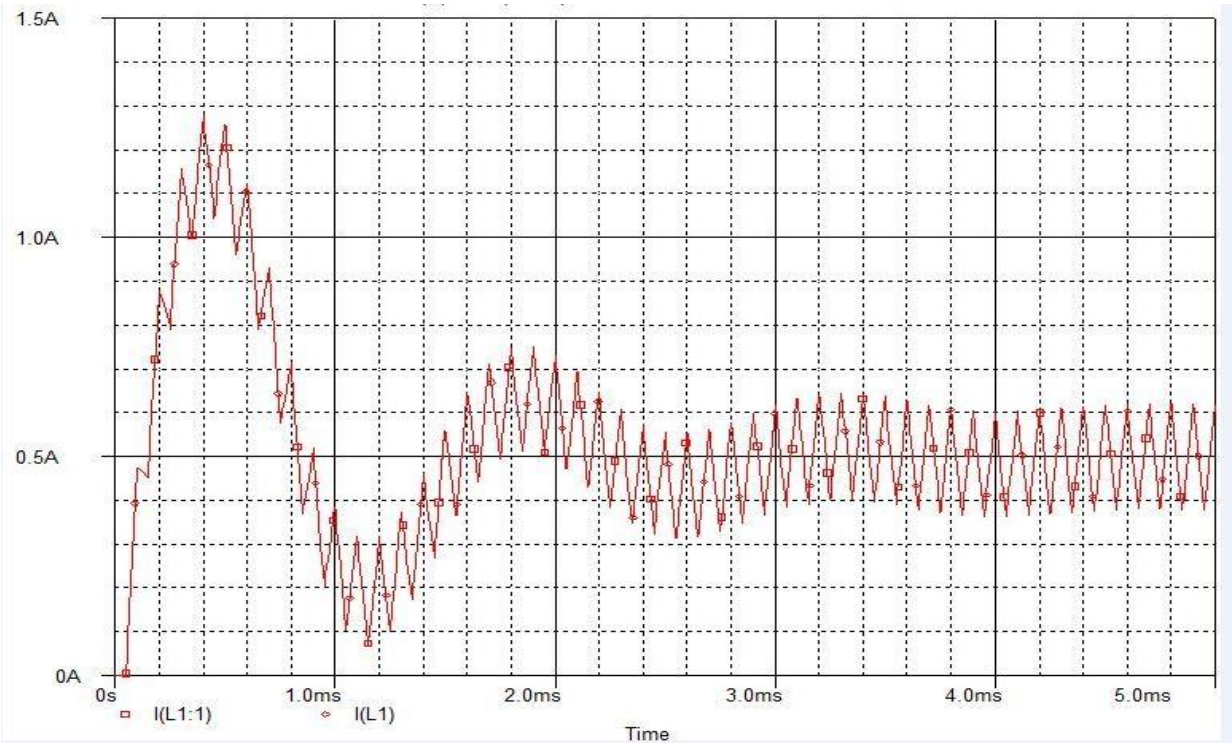


Figure 9 Waveform of inductor current(Y axis) vs. time(X axis)

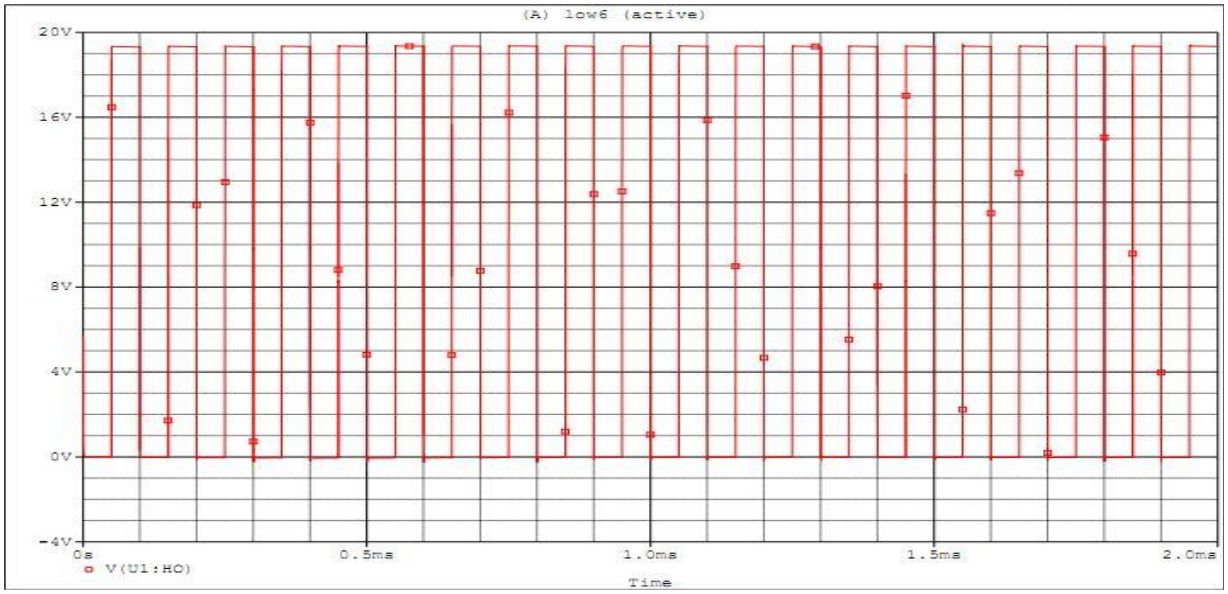


Figure 10 Waveform of high side MOSFET gate voltage(Y axis) vs. time(X axis)

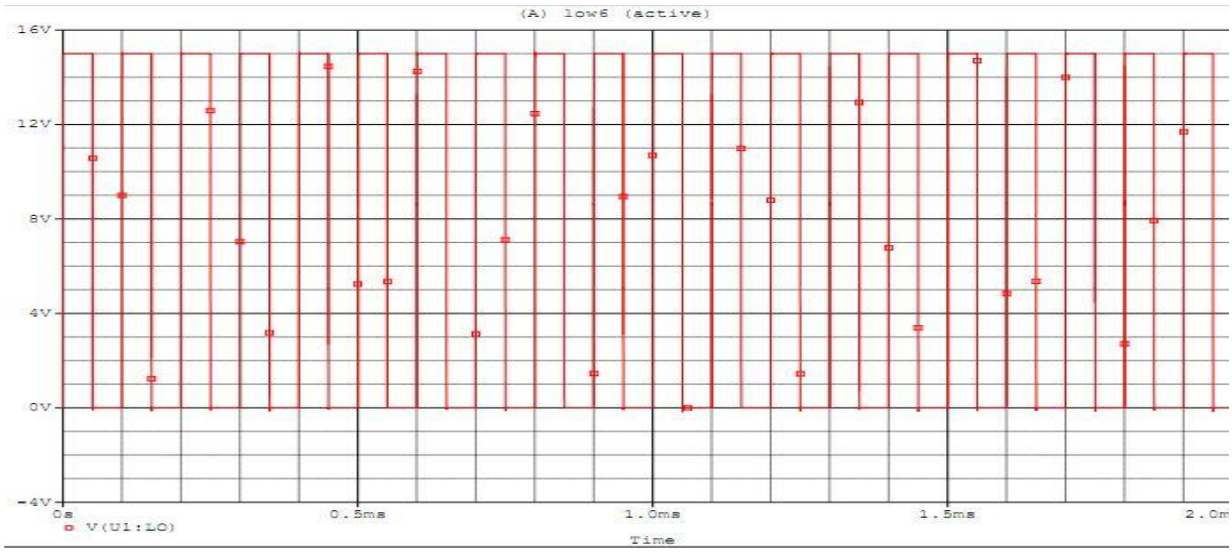


Figure 11 Waveform of low side MOSFET gate voltage (Y axis) vs. time(X axis)

2.7 Shoot through effect of MOSFET

The lower value of the Qgd/Qgs ratio of the low-side MOSFET, the better the synchronous buck will be to dV/dt shoot-through. To resolve this purpose the ratio should be <1. Ideal MOSFET is hardly feasible in real-life.as buck converter uses two MOSFET at any instant of time one MOSFET get turn on another get turn off. The MOSFET usually takes some time to get turn on and off may be in the order of ns but it takes. Hence there should have enough time delay between turning on of one MOSFET and turning off one MOSFET .Otherwise two MOSFET get turn off in a single time and the shoot through effect is take place which is not desire.

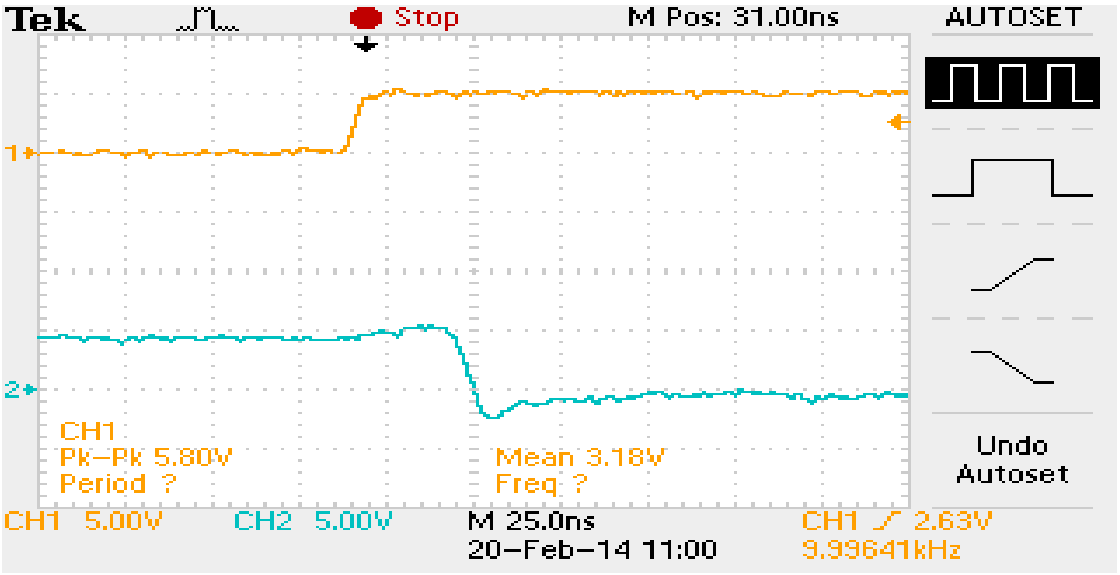


Figure 12 Delay between two PWM signals

2.8 Hardware model

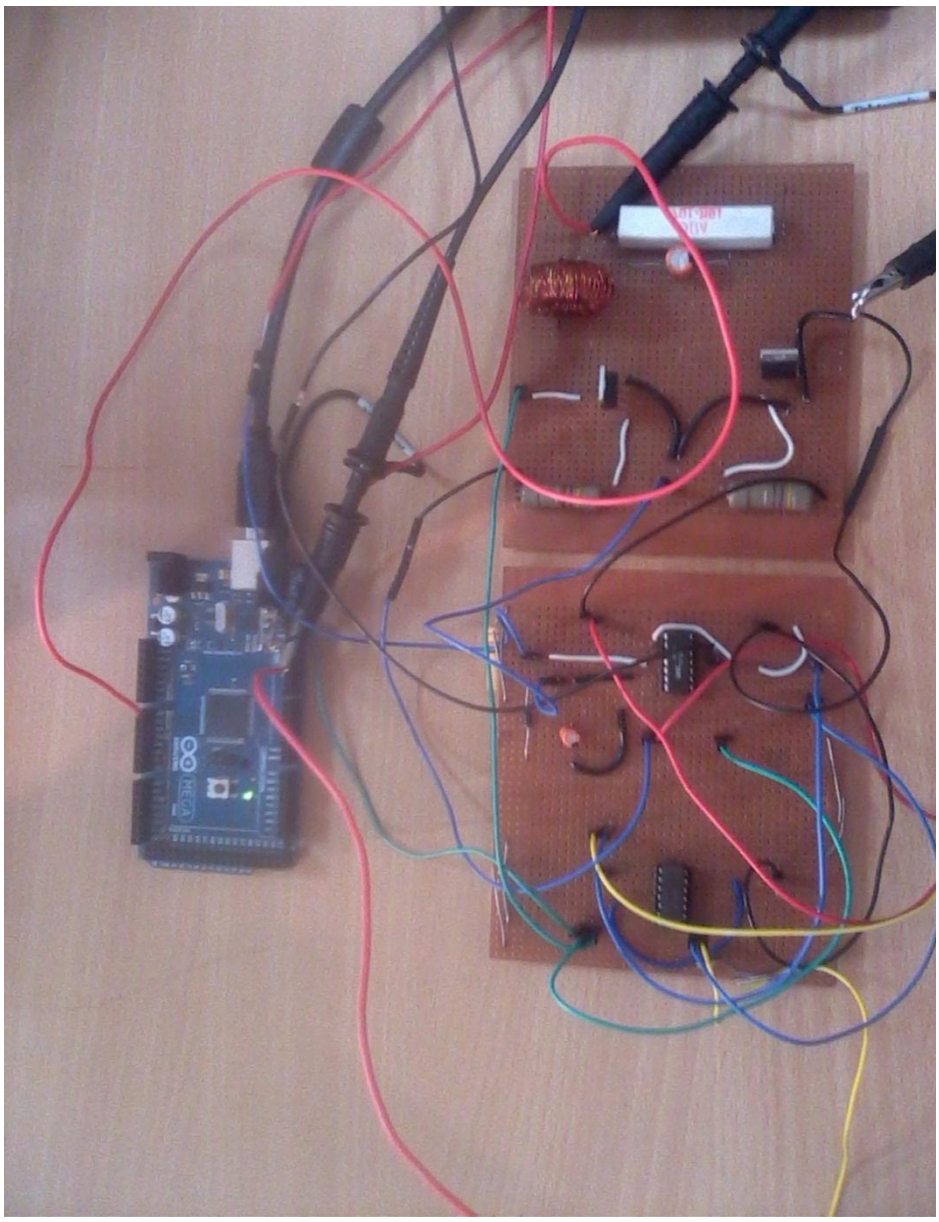


Figure 13 Hardware model

2.8 Hardware result

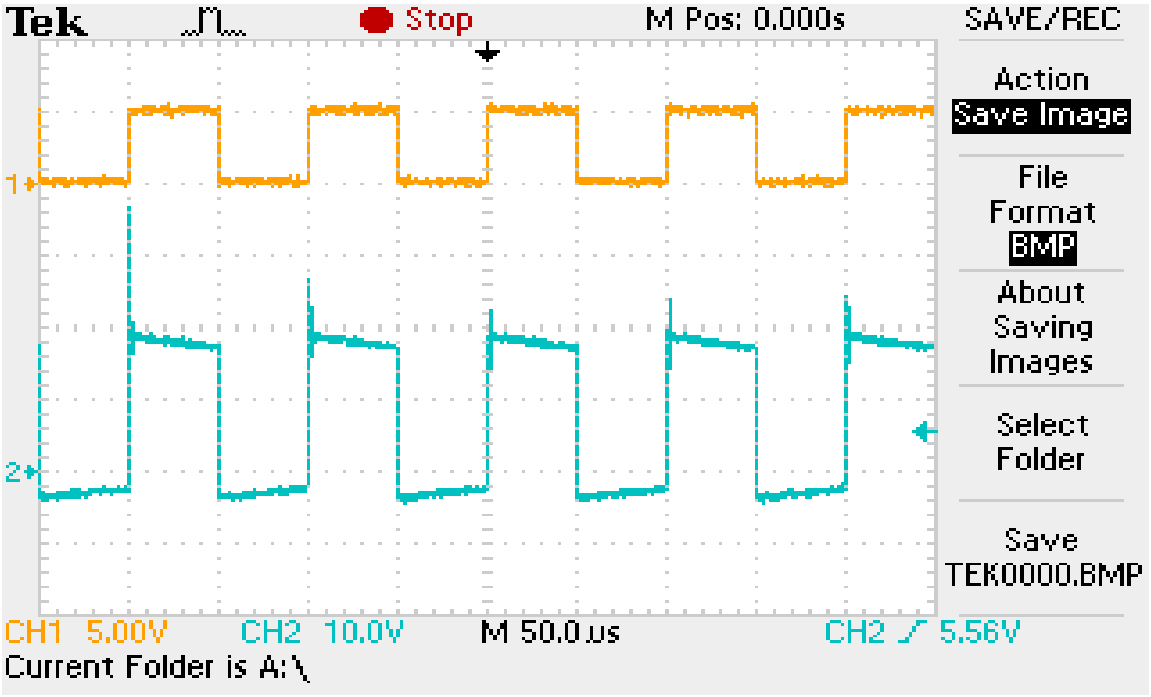


Figure 14 waveform of high side MOSFET gate voltage (Y axis) vs. time(X axis)

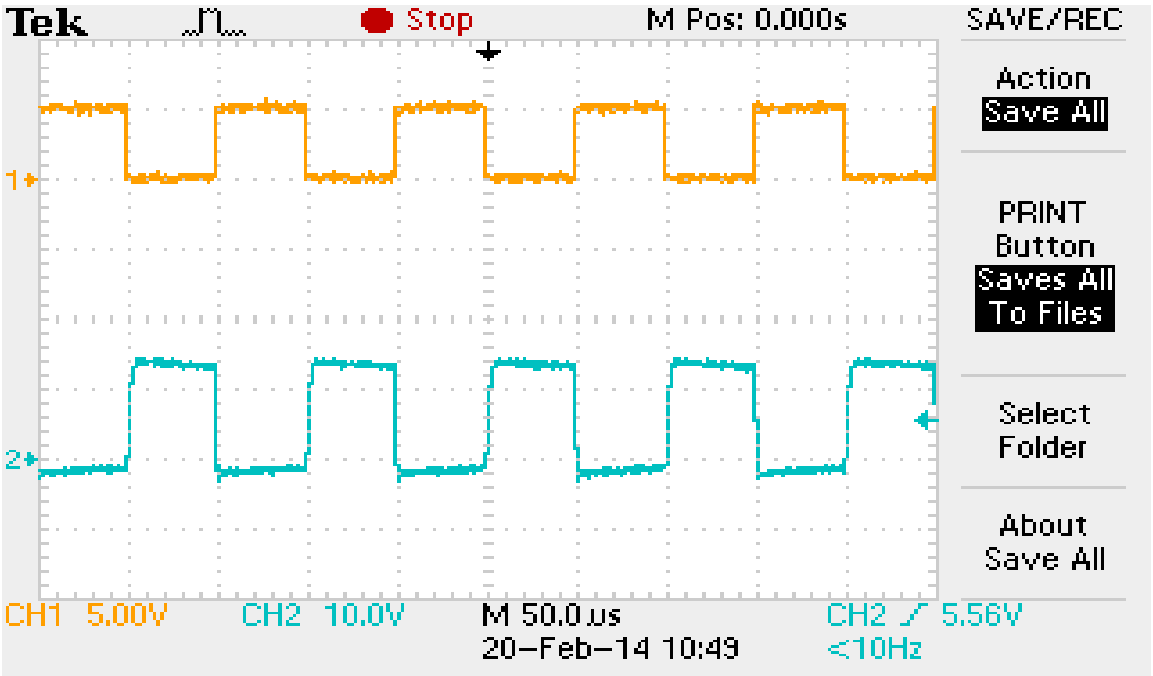


Figure 15 waveform of low side MOSFET gate voltage (Y axis) vs. time(X axis)

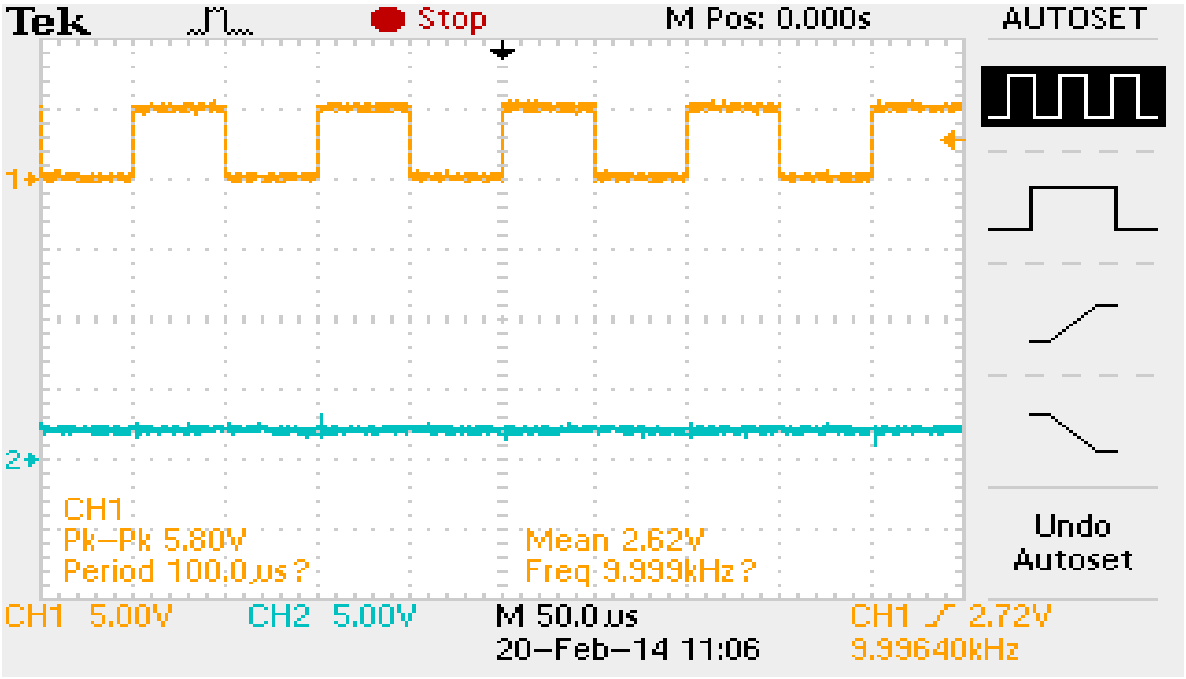


Figure 16 Waveform of output voltage (Y axis) vs. time(X axis)

Chapter 3

Pulse Width Modulation Techniques

In recent time it is seen that digital controller is far more advantageous over analog controller. For switching action of the MOSFET of the converter, high resolution digital pulse-width modulator (DPWM) is required in order to achieve appreciable output voltage regulation and to eliminate errors due to the quantization effects of the ADC and the DPWM. In recent years, several methods are proposed for PWM generation as explained in [1].

3.1 Pulse Width Modulation

Pulse width Modulation is the most commonly used technique for controlling the electric power to electric devices .For PWM voltage or current, supplied to the load can be varied according to switch “ON” or “OFF” state. For a higher “ON” state of the switched corresponds to high voltage or current which transfer a high power to the load. Duty cycle of a PWM signal is the measure of “ON” time of the signal so duty cycle is proportion of “ON” time to its period. In general duty cycle is expressed in the term of percentage so 100% denotes fully “ON” state.

The crucial point of PWM is that power dissipation in the switch in devices is very small during the “off” time there is practically no current flows through the switching device hence there is no voltage drop. And during its on time voltage drop across the switch is zero. As we know power loss is the product of voltage and current so in both cases power dissipation is zero. Because of its on-off nature it works very well in digital control. By using this PWM technique we can easily set a desired duty ratio.

The average value is given as-

$$Y_{avg} = \frac{1}{T} \int_0^T f(t)\ \mathrm{dt}$$

Assuming the pulse waveform has low value Y_{min} and high value Y_{max}

$$Y_{avg} = \frac{1}{T}\left(\int_0^{D.T} f(t)\ \mathrm{dt} + \int_{D.T}^{T} f(t)\ \mathrm{dt}\right) \quad (3.1)$$

$$Y_{avg} = \mathrm{D}^* Y_{max} + (1\text{-}\mathrm{D})\ Y_{min} \quad (3.2)$$

If $Y_{min} = 0$ then $Y_{avg} = \mathrm{D}^* Y_{max}$

Hence it can be concluded that a signal is directly dependent on duty ratio.

There is two type of methods by which we can generate PWM signal one is analog method another is digital method. In this project only digital method of PWM generation with microcontroller is discussed.

3.2 Microcontroller based technique:

3.2.1 Microcontroller

Microcontroller used microprocessor as its CPU, in addition with fixed amount of RAM .ROM, input output port, Timer, Serial port, Interrupt and clock circuit embedded on a single chip. Microcontroller is the device that is usually dedicated to a particular application. For example TV remote control, mobile phone. Since microcontroller is powerful digital processor the degree of control and programmability they provide significantly enhance the effectiveness of the application. The 8051 is the first microcontroller of MCS-51 family introduce by Intel Corporation at the end of 1970.8051 is an 8 bit processor with 64 kb ROM and 128 byte RAM along with two timer, one

serial port, Six interrupt sources. The 8051 become widely popular after Intel allowed the other manufacturer to make a market any flavors of 8051.They please with the condition that they remain code-compatible with the 8051[10].

There are mainly two general programming approaches for generating PWM signal popularly used that are explained below.

- Sequential approach Technique
- Time multiplexed Technique

3.2.2 Sequential approach technique

In this approach microcontroller core processor monitors the timing generation for "ON" and "OFF" status. The algorithm and flowchart for this approach is explained below. Flowchart given below shows the algorithm used in this approach. The program flow is sequential and processor of 8051 is engaged in the time slot generation for PWM signal. The input to this program is the digitized error signal generated from the plant. Digital version of the error signal is generated from the A to D converter used prior to this system.

The program can be explained as the input in the form of 8 bits is obtained from the plant and then T_{on}, T_{off} timing values are calculated depending on the error. These timing values are fed to timer of the 8051 controller and the processor of the 8051 monitors these values and make the PWM output from the 8051 high and low representing "ON" and "OFF" status of the pin. The algorithm for this method is in flowchart of fig 3.5. Drawbacks of this method are the processor is engaged in the time slot generation which can be done on other way and hence this method is not efficient.

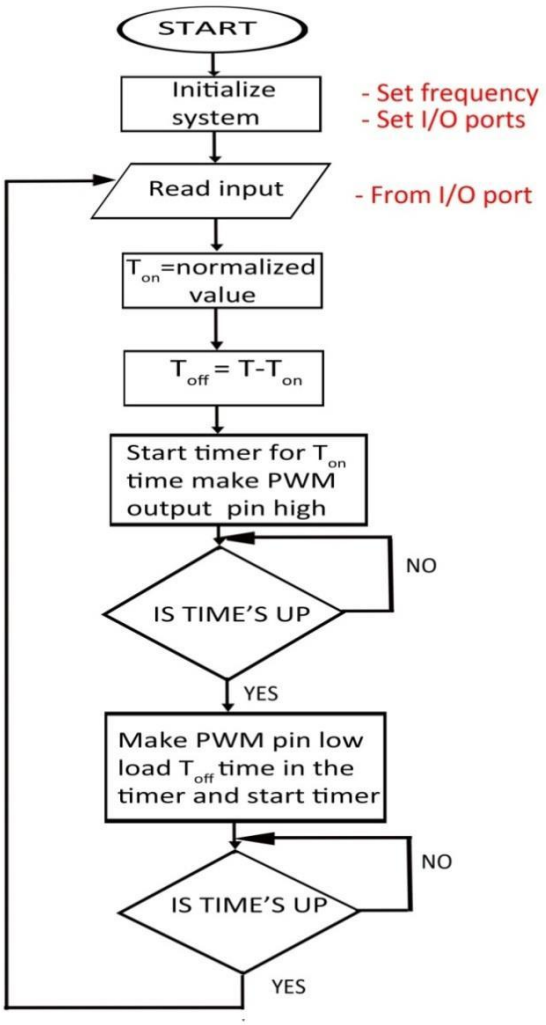


Figure 17 Flowchart for sequential approach

3.2.3 Time multiplexed approach:

For this approach, the concept of interrupts is exploited. Interrupts are explained as follows.

The time multiplexed approach is the proposed approach in for PWM generation in this thesis. This approach uses timer and external interrupt of 8051. These interrupts will produce an interfere to current process and make the processor to respond it. To respond the interrupt the microcontroller executes an interrupt handler program which is also called as Interrupt Service Routine (ISR). At this location the respective instructions are saved which will be executed when

the processor transfers the program control here.

Here at the ISR of timer 0, instructions for toggling the status of PWM output depending on T_{on} and T_{off} are saved and at the ISR of external interrupts a small program is written to update the T_{on} and T_{off}. The flowchart which is given below will explain the algorithm in detail.

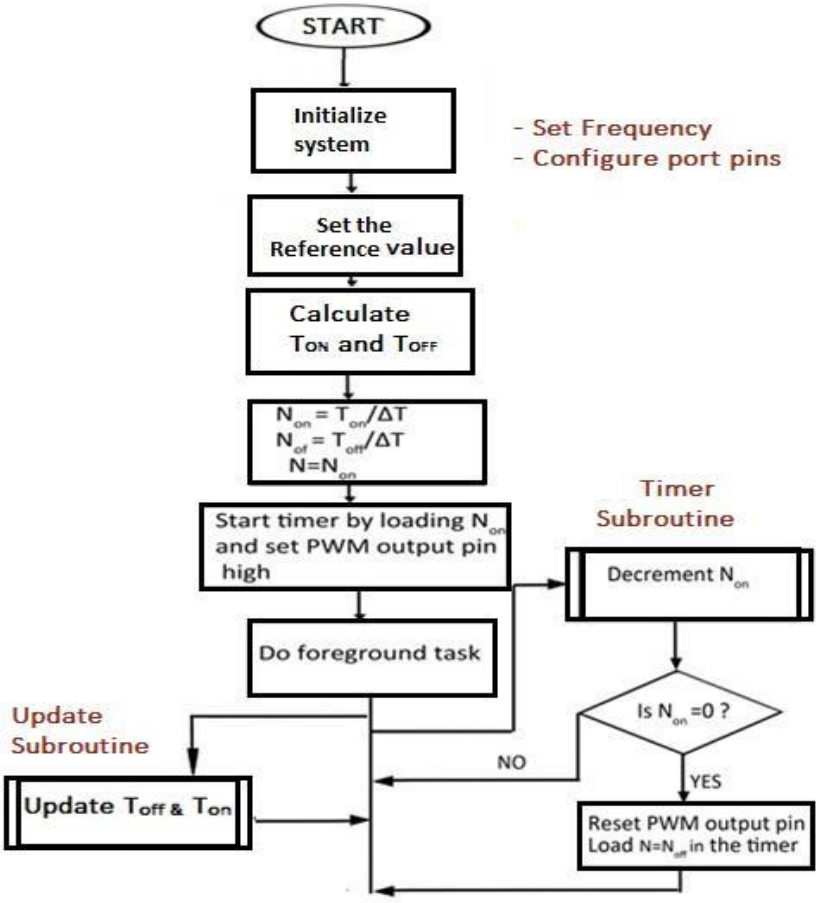


Figure 18 Flowchart for time multiplexed approach

3.3 NI MULTISIM Simulation diagram closed loop buck converter using sequential approach technique

The closed loop model of SBC using above listed DPWM technique is shown below.

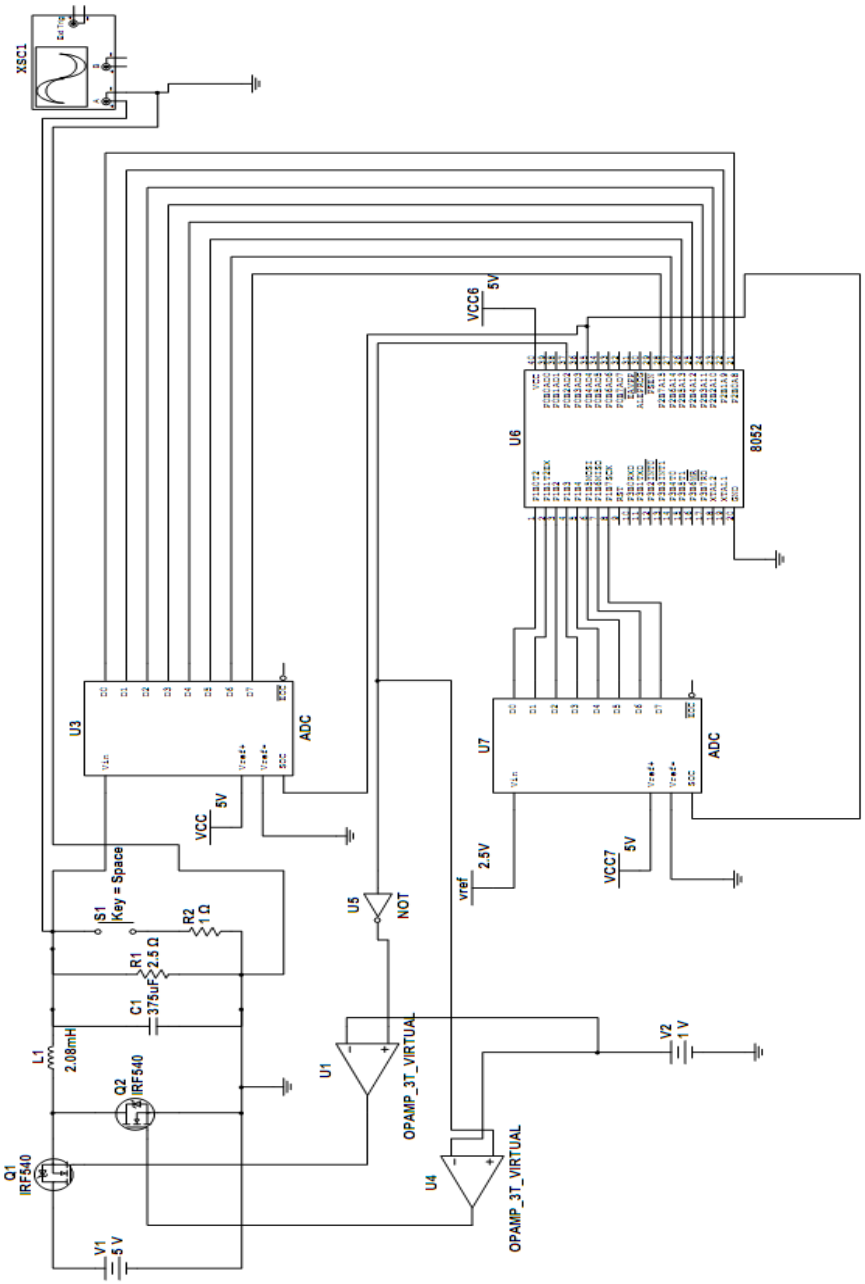


Figure 19 NI MULTISIM Simulation diagram closed loop SBC using 8052

3.4 Simulation result

The simulation result of closed loop synchronous buck converter with 5V applied voltage and 50% duty cycle is shown below.

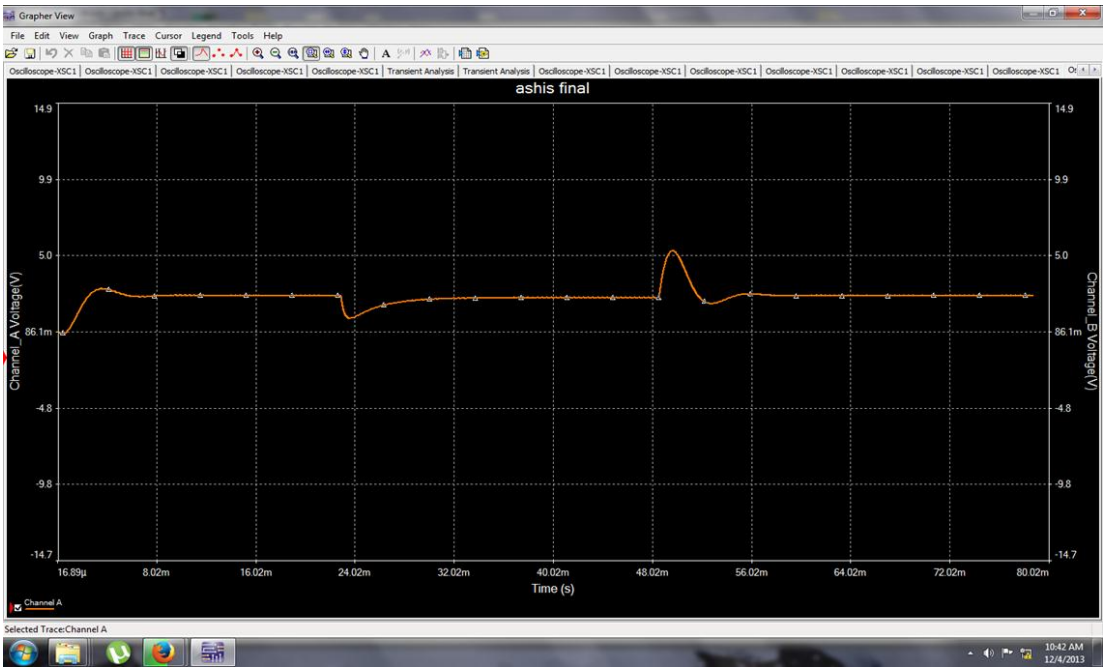


Figure 20 simulation result with load disturbance

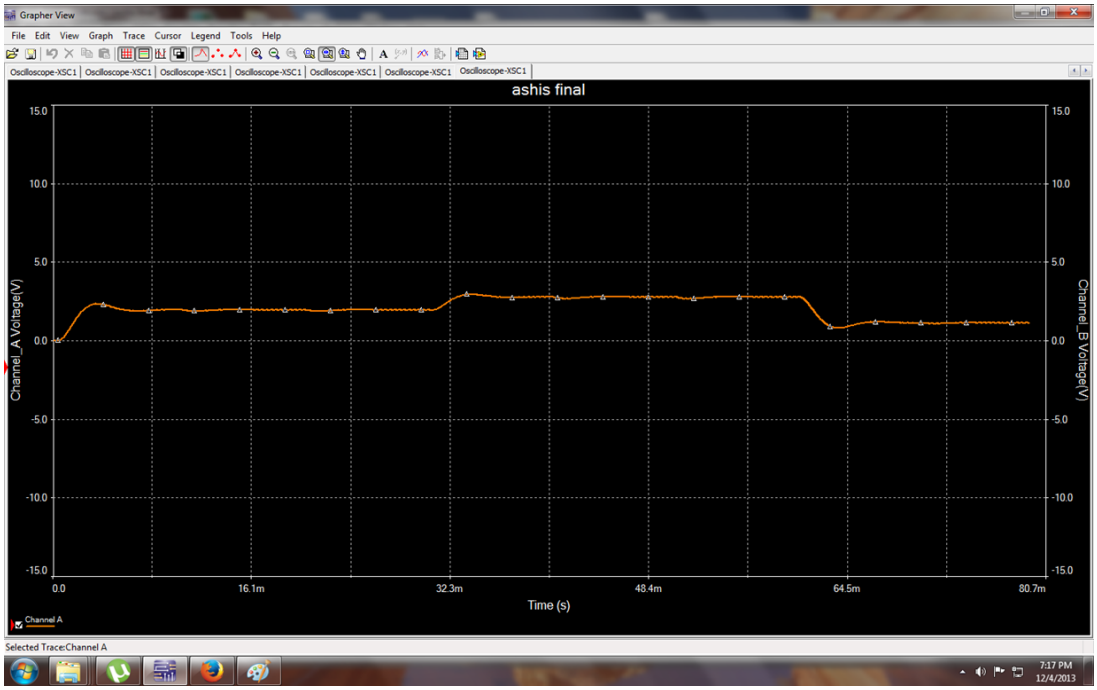


Figure 21 reference voltage tracking

Chapter 4

PID controller design

Usage of computerized PID controllers for DC-DC converters is talked about in this chapter. The essential preferences of digital control over analog control are higher invulnerability to ecological changes, for example, temperature and maturing of parts, expanded adaptability by changing the product, more praiseworthy control procedures and lessened number of segments. Simple PID and PI controllers were initially planned utilizing frequency response techniques, and then changed over into digital controllers.

4.1 Small Signal Analysis of Buck Converter

Small signal analysis is carried out to know the dynamics of the system and configuration the compensators for the switching converters. The small signal models incorporate different transfer functions, for example, control to o/p, o/p impedance and so forth. In this way we can design the compensator as per our desire with respect to any of these normal for transfer function. The principle reason for doing small signal analysis is to see the real behavior of the switching converter around a fixed operating point.

There are various methods that model these time variant systems into linear time invariant systems. State space averaging, Circuit averaging, Current injected approach are some of them. For our analysis we will take into account only state space averaging technique.

4.2 State Space Description for Each Interval

Here it is assumed that the buck converter is in continuous conduction mode. Therefore two circuits are considered, one for the on time and other for the off time of the converter. During T_{on} the switch is on and supply is connected to load through the inductor as shown.

So for on time our equation for inductor voltage and capacitor current are,

$$v_l(t) = v_g(t) - v_c(t) \quad (4.1)$$

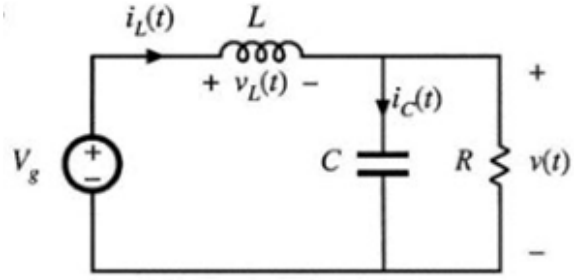
$$\mathbf{i_c(t) = i_l(t) - i_o(t)} \quad (4.2)$$

By expanding above equation we get,

$$\boldsymbol{L\frac{di_l}{dt} = v_g(t) - v_c(t)} \quad (4.3)$$

$$\boldsymbol{\frac{di_l}{dt} = \frac{v_g(t) - v_c(t)}{L}} \quad (4.4)$$

Similarly for the capacitor current,



$$\boldsymbol{C\frac{dv_c}{dt} = i_l(t) - i_o(t)} \quad (4.5)$$

$$\boldsymbol{\frac{dv_c}{dt} = \frac{i_l(t)}{C} - \frac{v_c(t)}{RC}} \quad (4.6)$$

Now the above equations can be written as,

$$\begin{bmatrix} \frac{di_l}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ -1/C & -1/RC \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} -1/L \\ 0 \end{bmatrix} \begin{bmatrix} v_g \end{bmatrix} \quad (4.7)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_l \\ v_c \end{bmatrix} \tag{4.8}$$

They may be written as,

$$\dot{x} = A_{on}x + B_{on}u \tag{4.9}$$

$$y = C_{on}x + D_{on}u \tag{4.10}$$

Here, D_{on} =0

Now we analyze the off time circuit,

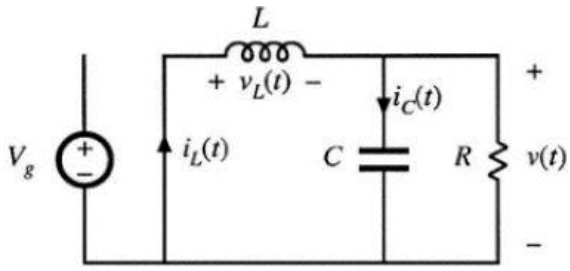
During off time the switch is open and the load current is supplied by the inductor stored energy.

And this path is completed through the diode.

During this time inductor voltage is,

$$\boldsymbol{v_l(t) = -v_c(t)} \tag{4.11}$$

$$\boldsymbol{\frac{di_l}{dt} = -\frac{v_c(t)}{L}} \tag{4.12}$$



Similarly for capacitor current,

$$i_c(t) = i_l(t) - i_o(t) \tag{4.13}$$

$$\frac{dv_c}{dt} = \frac{i_l(t)}{C} - \frac{v_c(t)}{RC} \tag{4.14}$$

These can also be written as,

$$\begin{bmatrix} \frac{di_l}{dt} \\ \frac{dv_c}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1/C & -1/RC \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} v_g \end{bmatrix} \tag{4.15}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_l(t) \\ v_c(t) \end{bmatrix} \tag{4.16}$$

Above equations can be rewritten as,

$$\dot{x} = A_{off} x + B_{off} u \tag{4.17}$$

$$y = C_{off} x + D_{off} u \tag{4.18}$$

Here $D_{off} = 0$

From above equations we can see that $A_{on} = A_{off}$ and $B_{off} = 0$

4.3 State Space Averaging

Let the converter switch be on for the time $d_n T$ i.e. t_{on} and $d_n^{'} T$ is the interval for which switch is off i.e. $t_{off} = d_n^{'} T = (1 - d_n) T$.

The state space description can be written as

$$\text{ON: } \dot{x}(t) = A_{on} x(t) + B_{on} u(t) \text{ for time } t \in \left[nT, nT + d_n T \right], \text{ n=1, 2, 3…} \tag{4.19}$$

OFF: $\dot{x}(t) = A_{off}x(t) + B_{off}u(t)$ for time $t \in [nT + d_nT, (n+1)T]$ n=1, 2,3… (4.20)

The solutions to the above equations can be found by taking the integration over each interval of operation,

$$x(n+d_n)T = e^{A_{on}d_nt}x(nT) + A_{on}^{-1}(e^{A_{on}d_nt} - I)B_{on}u \tag{4.21}$$

$$x(n+1)T = e^{A_{off}d_n'T}x(nT + d_nT) + A_{off}^{-1}(e^{A_{off}d_n'T} - I)B_{off}u \tag{4.22}$$

Substituting (3) in (4)

$$x(n+1)T = \left(e^{A_{off}d_n'T}\left(e^{A_{on}d_nt}x(nT) + A_{on}^{-1}(e^{A_{on}d_nt} - I)B_{on}u\right)\right) + A_{off}^{-1}(e^{A_{off}d_n'T} - I)B_{off}u$$

$$= e^{A_{on}d_nt}x(nT) + A_{on}^{-1}\left(e^{A_{off}d_n'T}e^{A_{on}d_nt} - e^{A_{off}d_n'T}\right)B_{on}u + A_{off}^{-1}(e^{A_{off}d_n'T} - I)B_{off}u \tag{4.23}$$

Then we get

$$x(n+1)T = e^{\left(A_{0n}d_n + A_{off}d_n'\right)T}x(nT) + A_{on}^{-1}\left(e^{A_{off}d_n'T}e^{A_{on}d_nt} - e^{A_{off}d_n'T}\right)B_{on}u$$

$$+ A_{off}^{-1}\left(e^{A_{off}d_n'T} - I\right)B_{off} \tag{4.24}$$

Now introducing the linear ripple approximation,

$$e^{AT} = I + AT$$

Applying this we can get,

$$x(n+1)T = x(nT) + \left(A_{on}d_n + A_{off}d_n'\right)Tx(nT) + d_nTB_{on}u + d_n'TB_{off}u \tag{4.25}$$

Expanding through Euler's approximation we can approximate derivatives as follows,

$$\dot{x} = \frac{x(nT+T) - x(nT)}{T}$$

Applying this to the equation (7) we get,

$$\dot{x} = \left(A_{on} d_n + A_{off} d_n^{'}\right) x + (B_{on} d_n + B_{off} d_n^{'}) u \tag{4.26}$$

$$y = \left(C_{on} d_n + C_{off} d_n^{'}\right) x + (E_{on} d_n + E_{off} d_n^{'}) u \tag{4.27}$$

Intrinsically the d is a discrete quantity with single value over a cycle. Therefore $d_n(t)$ can be replaced by *d(t)* assuming a very small variations occur that can be neglected.

$$\dot{x} = \left(A_{on} d(t) + A_{off} d^{'}(t)\right) x + (B_{on} d(t) + B_{off} d^{'}(t)) u \tag{4.28}$$

$$y = \left(C_{on} d(t) + C_{off} d^{'}(t)\right) x + (E_{on} d(t) + E_{off} d^{'}(t)) \tag{4.29}$$

4.4 Linearization

The equations derived above are non-linear and we have to linearize them. To linearize and obtain small signal model we have to perturb them around an operating point (D, X, U).

Let,

$$d = D + \hat{d}, x = X + \hat{x}, u = U + \hat{u}$$

(12)For small signal model we consider only first order terms and we obtain,

$$\hat{y} = C\hat{x} + E\hat{u} \tag{4.30}$$

Where $A = DA_{on} + D^{'}A_{off}$, $B = DB_{on} + D^{'}B_{0\,ff}$, $C = DC_{on} + D^{'}C_{off}$, $E = DE_{on} + D^{'}E_{off}$

The steady state value of duty ratio and state variables are obtained by considering the constant terms equal to zero and given as:

$$AX + BU = 0 \tag{4.31}$$

$$Y = CX + EU \tag{4.32}$$

So using equation (14) & (15) we get,

$$Y = -CA^{-1}BU + EU \tag{4.33}$$

Now extracting the small signal model:

Applying Laplace transform to the equation and considering initial conditions to zero we get,

$$s\hat{x}(s) = A\hat{x}(s) + B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d} \tag{4.34}$$

$$(sI - A)\hat{x}(s) = B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d} \tag{4.35}$$

$$\hat{x}(s) = (sI - A)^{-1}\left[B\hat{u}(s) + ((A_1 - A_2)X + (B_1 - B_2)U)\hat{d}\right] \tag{4.36}$$

By solving for above equation we get control to output transfer function we get,

$$G_{vd} = \frac{v_0}{\hat{d}} = \frac{RV_g}{(s^2RLC + sL + R)} \tag{4.37}$$

If all the non-ideal case is considered, then during on time the converter circuit will be like this.

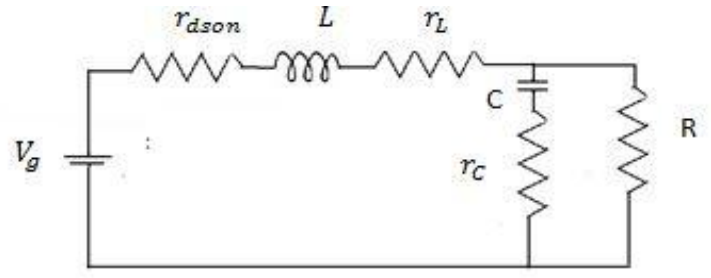


Figure 22 ON time circuit

The corresponding state space equation,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}(r_L + r_{dson} + \frac{Rr_c}{R+r_c} & -\frac{1}{L}\frac{R}{(R+r_c)} \\ \frac{R}{C(R+r_c)} & -\frac{1}{C(R+r_c)} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & -\frac{r_c R}{L(R+r_c)} & 0 \\ 0 & \frac{R}{C(R+r_c)} & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{4.38}$$

$$\begin{bmatrix} v_0 \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{Rr_c}{R+r_c} & \frac{R}{R+r_c} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{r_c R}{R+r_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \tag{4.39}$$

When the switch is off the circuit of converter shown below

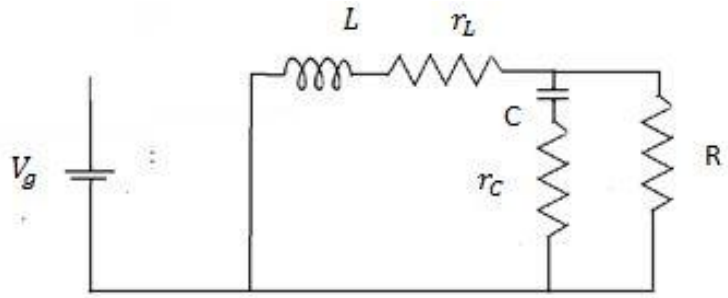


Figure 23 off time circuit

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{1}{L}(r_L + r_d + \frac{Rr_c}{R+r_c} & -\frac{1}{L}\frac{R}{(R+r_c)} \\ \frac{R}{C(R+r_c)} & \frac{1}{C(R+r_c)} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & -\frac{r_c R}{L(R+r_c)} & -\frac{1}{L} \\ 0 & \frac{R}{C(R+r_c)} & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \quad (4.40)$$

$$\begin{bmatrix} v_0 \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{Rr_c}{R+r_c} & \frac{R}{R+r_c} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 & \frac{r_c R}{R+r_c} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_g \\ i_{load} \\ v_d \end{bmatrix} \quad (4.41)$$

By using state space averaging as done in above ideal case and after linearizing the equation it can be write that,

$$V_0 = \frac{Rr_c}{R+r_c} I_L + \frac{R}{R+r_c} V_c \quad (4.42)$$

Where, $I_L = \frac{D_p V_g}{a} - \frac{(1-D_p)V_d}{a}$ and $V_c = \frac{RD_p V_g}{a} - \frac{R(1-D_p)V_d}{a}$ (4.43)

By putting the value of I_L and V_c in equation no. (5)

$$V_0 = \frac{Rr_c}{R+r_c} \times \frac{D_p V_g}{a} - \frac{Rr_c}{R+r_c} \times \frac{(1-D_p)V_d}{a} + \frac{R}{R+r_c} \times \frac{RD_p V_g}{a} - \frac{R}{R+r_c} \times \frac{R(1-D_p)V_d}{a} \quad (4.44)$$

Now, $r_{on} = r_L + r_{dson}$ $r_{off} = r_L + r_d$ and $a = R + D_P r_{on} + D_P r_{off}$

So, for a given output voltage and input voltage the duty ratio with parasitics is derived as,

$$D_p = \frac{V_0(R + r_{off}) + V_d R}{V_0(r_{off} - r_{on}) + R(V_g + V_d)} \quad (4.45)$$

After obtaining steady state values of D_P , I_L and V_c, all the transfer function arising out of the state space model can be derived for open loop power stage. Assume that the state of the incremental linearizing model is zero initially.

Then the transfer function is,

$$G_{vd}(s) = \frac{R(V_g + V_d + I_L(r_{off} - r_{on}))(1 + sCr_c)}{\Delta(s)} \quad (4.46)$$

Where,

$$\Delta(s) = s^2 LC(R + r_c) + s\left[L + RC\left\{(r_c + r_{off}) + D_p(r_{on} - r_{off})\right\} + r_c C\left\{D_p(r_{on} - r_{off}) + r_{off}\right\}\right] + R + D_p r_{on} + D_p r_{off} \quad (4.47)$$

By putting all the value of parameter in equation no. the transfer function can be re written as,

$$G_{vd}(s) = \frac{6.014 \times e^{-5} s + 62}{5.066 \times e^{-7} s^2 + 5.734 \times e^{-4} s + 10.04} \quad (4.48)$$

4.5 Exact Tuning of the PID Controller

4.5.1 Motivation

The primary inspiration to utilize this method is that it serves to plan the controller on the mixture of the phase margin with bandwidth and gain margin with phase margin. It likewise serves to fuse the steady state performance. This technique likewise has the focal point of figuring the parameters by utilization of pen, paper and the calculator.it might be connected to the systems where the plants careful model is not known however a little learning of the plants bode response will bail us out of the issue..

4.5.2 Objective

To plan a controller that fulfills our requirements of desired frequency response characteristics in programming (MATLAB) and likewise in equipment additionally with a little blunder and alteration.

4.5.3 Method of tune

The loop transfer function should satisfy two main condition:

1. The transfer function should be proper and it should not contain any right pole.
2. The polar plot of the L($j\omega$) for ω≥0 intersects the unit circle and negative semi-real axis only once.
3. If this criteria is met then proceed to the method described below

   Transfer function of PID controller:

$$C_{PID} = K_P\left(1+\frac{1}{T_i s}+T_d s\right) \tag{4.49}$$

The equation can be written in polar form

$$C_{PID}(j\omega) = M(\omega)e^{j\varnothing(\omega)} \tag{4.50}$$

Also plant can be represented as

$$G(j\omega) = |G(j\omega)|\, e^{j\angle G(j\omega)} \tag{4.51}$$

Now loop transfer function becomes

$$L(j\omega) = |G(j\omega)|\, M(\omega)e^{j(\varnothing(\omega)+\angle G(j\omega))} \tag{4.52}$$

Now parameters will be calculated as follows:

$$K_P = M_g\cos(\varnothing_g) \tag{4.53}$$

$$T_i = \frac{tan\varnothing_g + \sqrt{tan^2\varnothing_g + 4\sigma}}{2\omega_g\sigma} \tag{4.54}$$

$$T_d = T_i\sigma \tag{4.55}$$

Here $M_g = M(\omega_g)$

$$\varnothing_g = PM - \pi - \angle G(j\omega)$$

- σ = degree of freedom= $\frac{T_d}{T_i}$ this ratio affects the position of zeroes of PID

4.5.4 Validation of above described method

The above is applied to the buck converter to verify whether the parameter values obtained from the previous method gives the satisfactory result or not.

Component values of the Buck converter:

Inductor (L) =.0.522mH, r_L= 0.02 Ω, Capacitor(C) =94µF, r_C= 0.01 Ω load resistance (R) = 5Ω, Switching Frequency=10 kHz

Input voltage=5V, Output voltage=2.5V, Output current=0.5A

We choose PM=60°, steady state gain = 30 dB ,$f_c = 1kHz$ and we get controller parameters as follows,

- For $\sigma^{-1} = 5$, $K_P = 0.175$, $K_I = 9196$, $K_D = 6.66\times10^{-5}$

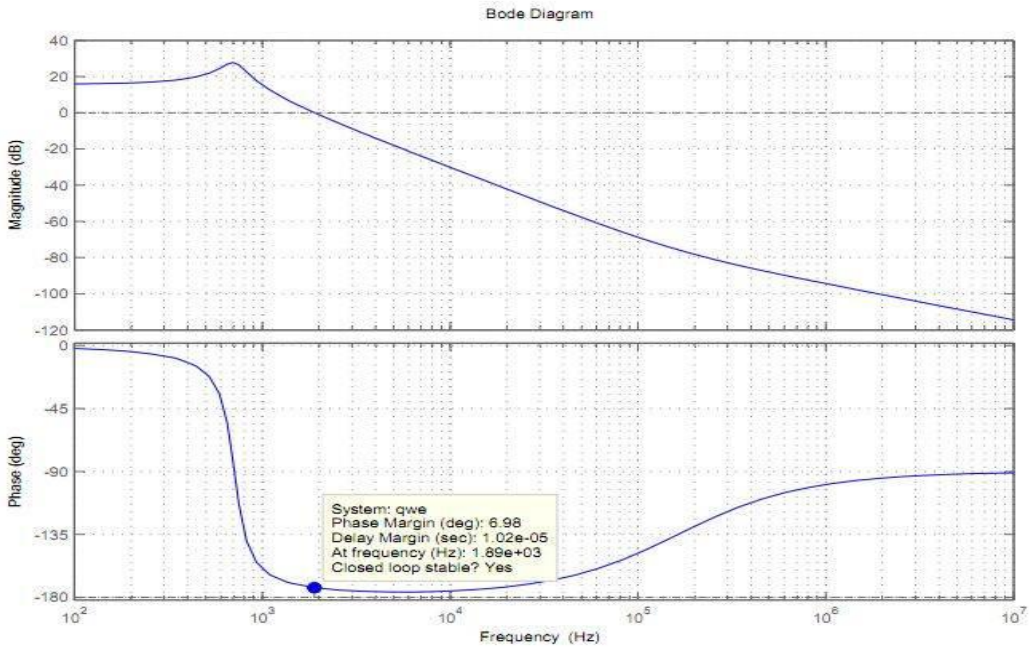


Figure 24 Bode plot of uncompensated system

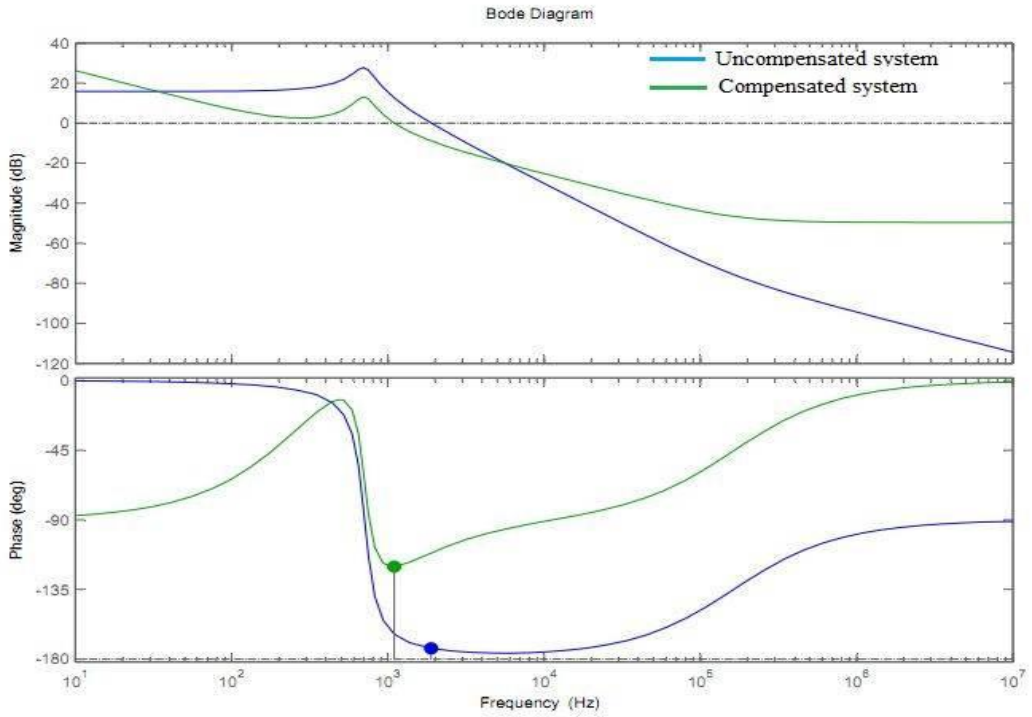


Figure 25 Bode plot of compensated system

Flowchart of PID algorithm in digital domain

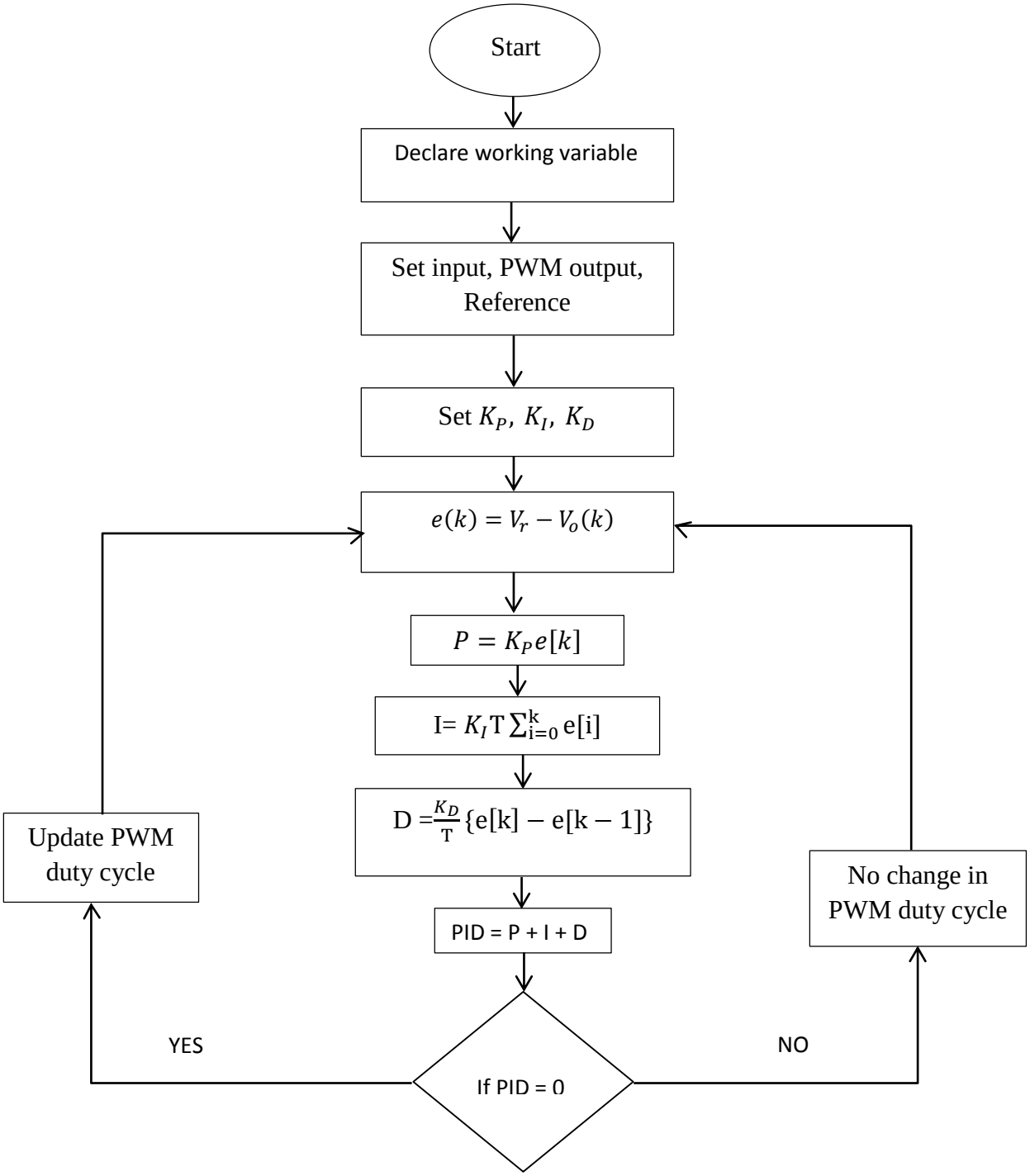


Figure 26 flow chart of digital PID controller

The transfer function of an integrator can be obtain by using backward difference method

$$M(z) = \frac{Tz}{z-1} E(z) \tag{4.56}$$

As the differentiator is the reciprocal of integrator transfer function. Hence,

$$G_c(z) = K_P + K_I \frac{Tz}{z-1} + K_D \frac{z-1}{Tz} \tag{4.57}$$

To calculate the duty cycle required difference equation for digital PID controller is,

$$u[k] = K_P e[k] + K_I T \sum_{i=0}^{k} e[i] + \frac{K_D}{T} \{e[k] - e[k-1]\} \tag{4.58}$$

4.6 Hardware model

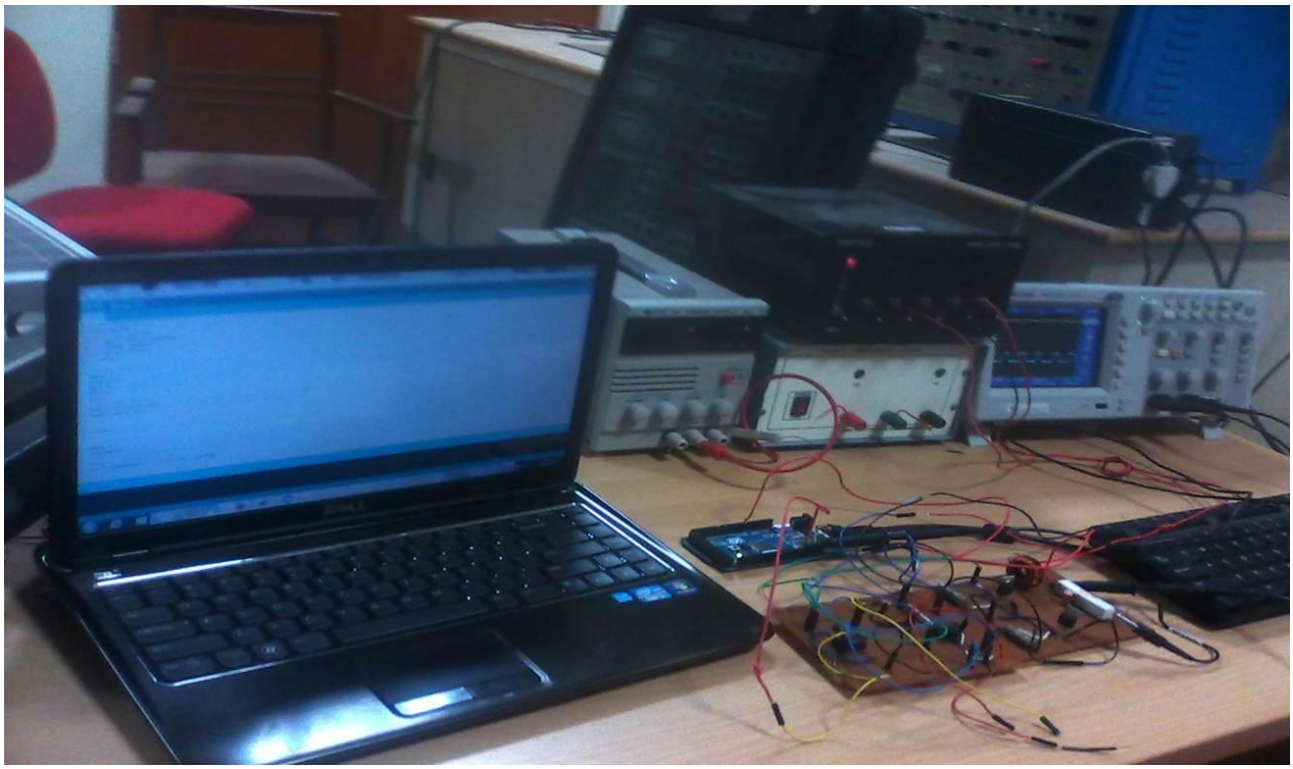


Figure 27 hardware circuit

4.7 Hardware result of different reference voltage

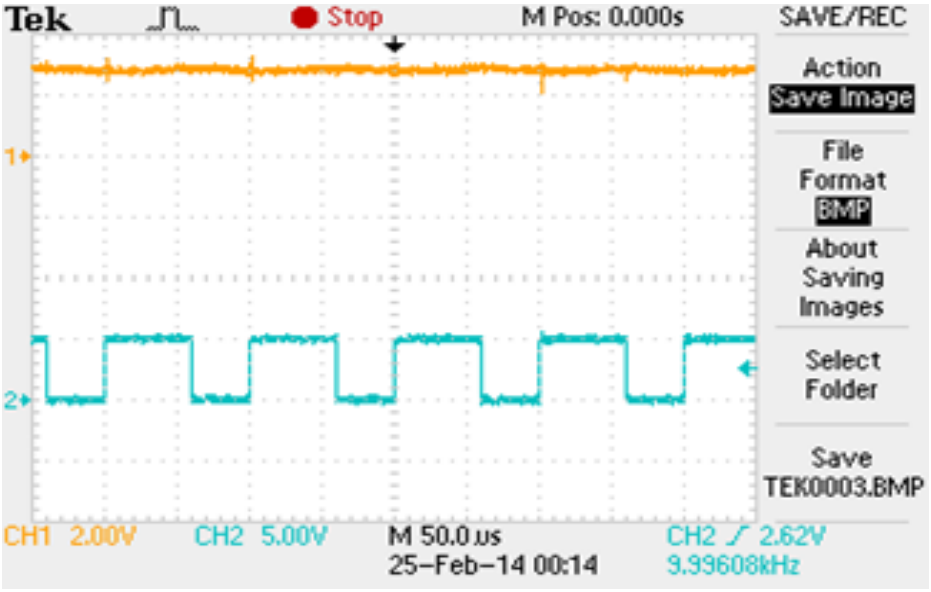


Figure 28 Hardware output voltage when reference voltage is 3V

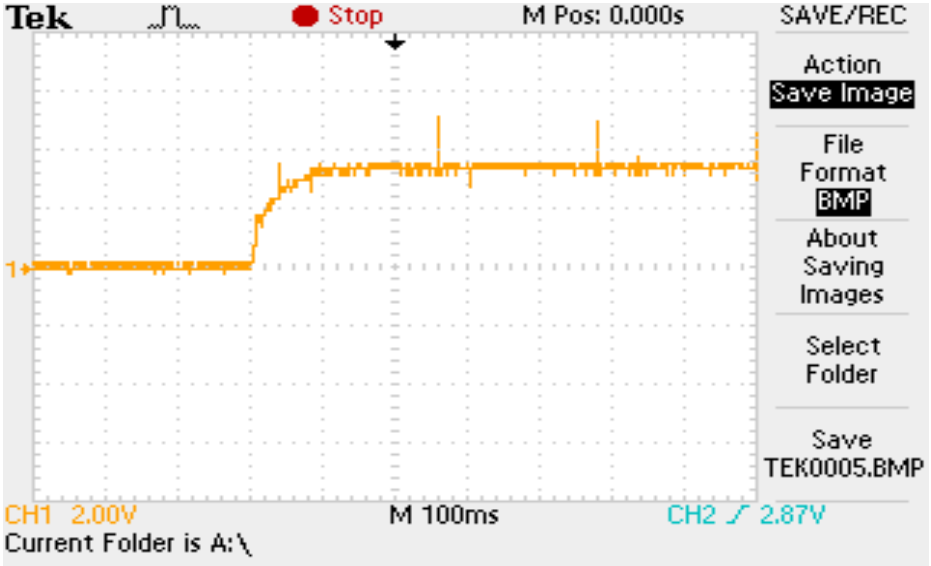


Figure 29 Startup transient responses when reference voltage is 3V

Chapter 5

Conclusion

DC-DC converters proselyte unregulated DC data voltage into controlled DC output voltage. Advanced digital control of DC-DC converters has a few preferences over simple analog control. The favorable circumstances incorporate higher resistance to ecological progressions, expanded adaptability by changing the software, more developed control methods, integration of power management function and diminished number of component.

Advanced PID is executed on the ARDUINO microcontroller for a model synchronous buck converter and trial results were exhibited. Exploratory results show that the digital PID and PI controllers could attain quick transient reaction without overshoot and great dismissal to load aggravations in relentless state.

The digital adaptive PID is model free since the gain factor $\beta(k)$ is only on the normalized error $e_N(k)$ and normalized error change $\Delta e_N(k)$

- It can be implemented to any other converter topology
- This scheme is easily be implemented using DSP or FPGA

5.1 Suggested Future work

The immediate future work would be to use a suitable state feedback controller for efficient controlling and make the system more stable

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